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ABSTRACT

The American College Testing Program (ACT) routinely evaluates the fit of its tests to the curriculum of high schools nationwide to ensure that what is being assessed reflects what is being taught as necessary for college success. This report describes the steps ACT has taken to ensure that the content foundations of the ACT Assessment Mathematics Test are solid. Evidence is provided from: (1) an account of how the content and skills measured by the ACT Assessment Mathematics Test are determined and how they are kept up to date; (2) survey results demonstrating that what the Mathematics Test measures is consistent with what is being taught in high schools that is necessary for college success; (3) reviews of state curriculum frameworks and national curriculum and performance standards supporting the validity of the test at the high school level; (4) a summary of ACT's review of current mathematics textbooks nationwide; (5) documentation of the role U.S. mathematics teachers play in developing the ACT mathematics test; (6) results of predictive validity and placement studies that show the mathematics test is effective in measuring the mathematical skills important for college success. (Author/PVD)

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Mathematics for a Successful Transition to College

The Content Foundations of the ACT Assessment

November 1996

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ACT and the ACT Assessment

ACT was founded in 1959 as a not-for-profit organization and has grown to become one of the leading providers of educational assessment and research services in the world. With its focus on providing information for life's transitions, ACT provides a wide range of services and programs in areas such as educational assessment, career planning, student financial aid, educational data management, and educational and psychometric research.

ACT's flagship program, the ACT Assessment Program, provides a wide range of assessment information and services to help students make a successful transition from high school to postsecondary education. Over 1.5 million tests are administered each year to college-bound students. The ACT Assessment tests include questions that measure problem-solving and critical-reasoning skills across the major curricular areas of English, mathematics, reading, and science reasoning. Materials taken from the humanities (arts, music, philosophy, theater, architecture), prose fiction (short stories, novels), social studies (history, political science, economics, anthropology, psychology, sociology), and natural sciences (biology, chemistry, physics, Earth/space science) serve as the contexts for the measurement of these skills. These materials ask students to solve problems, draw inferences, interpret information, evaluate ideas, and compare concepts—that is, to *apply* the knowledge and skills they have learned in major curriculum areas.

The ACT Assessment Program also includes a needs assessment and an interest inventory. The needs assessment allows students to identify academic areas in which they need some help, a lot of help, or no help. This provides both high school and college faculty with necessary information to help students be academically successful. The interest inventory permits students to identify areas of vocational interest so that the students can compare their interests, academic performance, and career plans, to make sure they are complementary.

Students can also self-report important information about themselves. This information can be used by colleges and other institutions to better understand and advise incoming students.

Information from the ACT Assessment is provided to students and high schools, as well as to colleges designated by the student. Based on this information and other information provided by the high school, such as high school grade point average and a course transcript, decisions can be made by colleges about admissions and course placement. Similarly, ACT Assessment information can be used by colleges and other institutions to make scholarship awards. The ACT Assessment Program offers a well-rounded picture of the college applicant, thus enhancing the student's chances for success in making the transition from high school to college.

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Introduction

The ACT Assessment Program was established by E. F. Lindquist, who believed that the test should measure what students can do with what they have learned throughout their schooling. Further, this measurement needed to be grounded in the kinds of courses students typically take in high school that are important for college success. Dr. Lindquist's philosophy has guided the specifications for the ACT Assessment educational achievement tests, both when they were created and today.

The ACT Assessment Mathematics Test is designed to assess the mathematical skills that students have typically acquired in courses taken up to the beginning of grade 12 and that are relevant to success in college. The test is one of four given as part of the ACT Assessment Program; all four tests have been designed to reflect today's curriculum. The ACT Assessment is the *only* curriculum-based college admissions and placement test available in the United States today that focuses on what is important for successful high school-to-college transitions.

ACT routinely evaluates the fit of its tests to the curriculum to ensure that what is being assessed reflects what is being taught in classrooms nationwide that is necessary for college success. This report describes the steps ACT has taken to ensure that the content foundations of the ACT Assessment Mathematics Test are solid. Evidence is provided from these sources:

- an account of how the content and skills measured by the ACT Assessment Mathematics Test are determined and how they are kept up to date,
- survey results demonstrating that what the ACT Assessment Mathematics Test measures is consistent with what is being taught in high schools today that is necessary for college success,
- reviews of state curriculum frameworks and national curriculum and performance standards supporting the validity of the ACT Assessment Mathematics Test at the high school level,
- a summary of ACT's review of current mathematics textbooks nationwide,
- documentation of the role that U.S. mathematics teachers play in developing the ACT Assessment Mathematics Test, and
- results of predictive validity and placement studies that show the ACT Assessment Mathematics Test is effective in measuring mathematical skills important for success in college.

Test Specifications Development

Test specifications are the blueprints that are used to construct a test. They are composed of two basic parts: (1) a description of the content and skills to be measured by the test and (2) a description of the statistical characteristics of the test questions. This report focuses on the content and skill specifications.

The original content specifications for the ACT Assessment Mathematics Test, like those of the other three ACT Assessment tests, were determined through a detailed analysis and review of the curriculum. To maintain the accuracy of the specifications and to monitor the secondary-postsecondary curriculum balance, ACT routinely repeats a similar analysis and review process. Test specifications are then refined or modified as appropriate. ACT gathers information in the following ways.

- High school teachers and college faculty are consulted to determine the knowledge and skills taught up to grade 12 that are required for successful performance in mathematics in college. This information, in survey form, provides lists of currently taught topics upon which to base the test specifications.

- The curriculum frameworks for grades 7 through 12 are obtained for all states in the United States that have published such frameworks. This information is used to identify and add any areas of the curriculum that were not covered in the survey process. These frameworks are reviewed and combined with the results of the surveys.

- Textbooks on state-approved lists for courses in grades 7 through 12 are reviewed. ACT uses the information from the textbooks to round out the topic list of what is being taught in high school mathematics courses. The textbook review also aids in understanding how students are being prepared for successful college mathematics performance.

These three sources of information are then analyzed to define a scope and sequence for mathematics learning.

ACT also tries to project how the curriculum may evolve. By reviewing materials such as the National Council of Teachers of Mathematics (NCTM) *Standards*, the National Assessment of Educational Progress (NAEP) mathematics framework, and other national efforts related to Goal 5 ("By the year 2000, United States students will be the first in the world in mathematics and science achievement") of the federal Goals 2000 initiative, ACT identifies emerging curricular trends that are likely to have an impact on the ACT Assessment.

The information ACT collects on the current and emerging curricular trends is then shared with panels of mathematics experts, including teachers, curriculum specialists, state department of education specialists, and college faculty in mathematics and education. Their reviews aid ACT in confirming the appropriateness of refinements to the specifications.

The results of all these analyses and reviews are carefully evaluated, resulting in the test specifications for the ACT Assessment Mathematics Test. The primary consideration in building the specifications is to maintain a delicate balance between what is being taught in the high schools and what are necessary skills for success in college-entry course work.

The ACT Assessment Mathematics Test

Table 1 provides a summary of the content specifications for the ACT Assessment Mathematics Test. The test presents multiple-choice questions that require students to use their reasoning skills to solve practical problems in mathematics. The problems assume knowledge of basic formulas and computational skills but do not require complex formulas nor extensive computations. The material covered on the test emphasizes the major content areas that are prerequisite to successful performance in entry-level courses in college mathematics.

Table 1
ACT Assessment Mathematics Test Content Specifications

Content Area	Number and Percentage of Questions
Pre-Algebra	14 (23%)
Elementary Algebra	10 (17%)
Intermediate Algebra	9 (15%)
Coordinate Geometry	9 (15%)
Plane Geometry	14 (23%)
Trigonometry	4 (7%)
Total Questions	**60**

- *Pre-Algebra.* Questions in this content area are based on basic operations using whole numbers, decimals, fractions, and integers; place value; square roots and approximations; the concept of exponents; scientific notation; factors; ratio, proportion, and percent; linear equations in one variable; absolute value and ordering numbers by value; elementary counting techniques and simple probability; data collection,

representation, and interpretation; and understanding simple descriptive statistics.

- *Elementary Algebra.* Questions in this content area are based on properties of exponents and square roots, evaluation of algebraic expressions through substitution, using variables to express functional relationships, understanding algebraic operations, and solving quadratic equations by factoring.

- *Intermediate Algebra.* Questions in this content area are based on an understanding of the quadratic formula, rational and radical expressions, absolute value equations and inequalities, sequences and patterns, systems of equations, quadratic inequalities, functions, modeling, matrices, roots of polynomials, and complex numbers.

- *Coordinate Geometry.* Questions in this content area are based on graphing and the relations between equations and graphs, including points, lines, polynomials, circles, and other curves; graphing inequalities; slope; parallel and perpendicular lines; distance; midpoints; and conics.

- *Plane Geometry.* Questions in this content area are based on the properties and relations of plane figures, including angles and relations among perpendicular and parallel lines; properties of circles, triangles, rectangles, parallelograms, and trapezoids; transformations; the concept of proof and proof techniques; volume; and applications of geometry to three dimensions.

- *Trigonometry.* Questions in this content area are based on understanding trigonometric relations in right triangles; values and properties of trigonometric functions; graphing trigonometric functions; modeling using trigonometric functions; use of trigonometric identities; and solving trigonometric equations.

Although the current test specifications do not specifically list "probability and statistics" as a separate category, questions in this area are included on the ACT Assessment Mathematics Test within other content categories such as Pre-Algebra. ACT is including more of these questions as the topic becomes more common in mathematics classrooms and increases in importance for success in college.

The questions included in the ACT Assessment Mathematics Test cover four cognitive levels: knowledge and skills, direct application, understanding concepts, and integrating conceptual understanding. Numbers and percentages of each type of question in the test are presented in Table 2.

- *Knowledge and Skills.* These questions require the student to use one or more facts, definitions, formulas, or procedures to solve straightforward problems that are presented in purely mathematical terms.

- *Direct Application.* These questions require the student to use one or more facts, definitions, formulas, or procedures to solve straightforward problems set in real-world situations.

- *Understanding Concepts.* These questions test the student's depth of understanding of major concepts by requiring reasoning from a concept to reach an inference or a conclusion.

- *Integrating Conceptual Understanding.* These questions test the student's ability to achieve an integrated understanding of two or more major concepts to solve nonroutine problems.

Table 2
ACT Assessment Mathematics Test Cognitive Level Specifications

Cognitive Level	Number and Percentage of Questions
Knowledge and Skills	32 (53%)
Direct Application	16 (26%)
Understanding Concepts/ Integrating Conceptual Understanding	12 (20%)
Total Questions	**60**

Beginning in fall 1996, students were permitted to use calculators on the ACT Assessment Mathematics Test. No changes in the test specifications were required to permit the addition of calculator use. ACT's research studies showed that the scale scores reported on the ACT Assessment Mathematics Test continue to have the same meaning as they had before calculators were permitted.

Updating Test Specifications

ACT's most recent national mathematics curriculum study to define and refine the test specifications was conducted in 1994–95. At that time, ACT reviewed the content specifications of all four academic tests in the ACT Assessment and their fit to the secondary-postsecondary curriculum continuum. The results of the reviews in mathematics are described in the following sections.

Curriculum Survey

In the spring of 1995, ACT conducted a mathematics curriculum survey targeted at 4,800 high school teachers, curriculum specialists, and department heads, as well as college faculty. Half of the 4,800 mathematics educators were from the secondary level and half from the postsecondary level. These groups were each further subdivided into equal groups—half responding to questions about various elements of probability and statistics, geometry, and trigonometry and half responding to questions about various elements of algebra and calculus. (The subdividing was required to make it possible to complete the survey in a reasonable amount of time.) All groups were asked about general mathematics skills. High school respondents were asked to rate the importance of the survey topics in high school mathematics courses. College and university respondents were asked to rate the importance of the topics as prerequisites for success in entry-level and advanced-level college mathematics courses. The response rate for each secondary group was 27%. The response rate for each postsecondary group was 16%.

Weighted numbers of respondents and their respective regions appear in Table 3.

Table 3
Weighted Number of Survey Responses by Geographic Region

Region	Weighted Sample	
	Secondary	Post-secondary
Pacific/Mountain	26	13
North Central	10	10
South Central	19	23
Southeast	10	15
Northeast	19	30
Great Lakes	26	29

All of the respondents were asked to consider the relative importance of mathematics content and skills that a typical student bound for a 4-year college needs to know. Secondary respondents were also asked the grade level at which the content or skill is first introduced to that student in the respondent's school or district. The surveys included topics in general mathematics skills, as well as the content areas of algebra, geometry, trigonometry, probability and statistics, and calculus. Space was provided for respondents to list other topics as desired. These added topics were subsequently reviewed by ACT staff and considered in the final

specifications review. Results of the responses were reported on a 5-point scale, where 1 = no importance, 3 = medium importance, and 5 = great importance. The responses from all secondary and all postsecondary respondents were combined for evaluation. The appendix contains a list of all survey topics and the mean ratings for each by respondent group.

The data described below are based on averages of the ratings of each topic by the respondents, with higher averages reflecting greater importance. Average ratings at or above 3 (medium importance) were tabulated.

General Mathematics Skills

The surveys listed 21 general mathematical skills. Of these 21 topics, college faculty reported that all but 6 of them (a total of 71%) were at least moderately important for entry-level freshmen to have mastered. High school teachers reported that all 21 topics were at least moderately important for college-bound students to know. Clearly, the kinds of general skills students acquire in their mathematics classes are important at both levels, high school and college.

The ACT Assessment reflects this emphasis, in that 14 of the topics are directly assessed on the Mathematics Test. Of the 7 topics ACT did not include on the Mathematics Test, 5 of them were rated below 3 for entry-level freshmen, indicating a high level of agreement between the test specifications and entry-level requirements in the area of mathematics skills. Those skills excluded from the ACT Assessment Mathematics Test specifications and rated lower than 3 for entry-level freshmen were: constructing an inductive proof, constructing a deductive proof, formulating new patterns or structures, performing statistical operations with a calculator, and graphing and exploring the nature of equations using a graphing calculator. Skills rated above 3 for entry-level freshmen but not directly assessed on the ACT Assessment Mathematics Test are communicating mathematically (orally) and communicating mathematically (in writing).

College faculty found 19 of the 21 topics, or 90%, to be important for advanced-level freshmen (rated at 3 or higher). The two topics rated lower were constructing an inductive proof and performing statistical operations with a calculator.

Algebra

A total of 46 algebra topics were listed in the survey. Of these topics, college faculty told ACT that 27 topics (59%) were at least moderately important. On the other hand, high school teachers reported that all but one of the topics (finding common factors and common multiples) were important from their perspective.

Of the 46 algebra topics included in the survey, 100% are included in the ACT Assessment Mathematics Test specifications. While the topic list encompasses more than what may be needed for entry-level freshmen, all but 4 of the topics on the algebra list were included as needed for advanced-level freshmen. These results indicate that the ACT Assessment Mathematics Test has a content basis in algebra for distinguishing between entry-level and advanced-level freshmen. This information can be useful in helping colleges and universities place incoming freshmen more effectively. The Mathematics Test produces 3 subscores: Pre-Algebra/Elementary Algebra, Intermediate Algebra/Coordinate Geometry, and Plane Geometry/Trigonometry. Algebra is an important component of two of the subscores, both of which can serve as useful bases for making placement decisions.

Geometry

There were 36 topics on the geometry list. College faculty told ACT that 16 of them (44%) were at least moderately important for entry-level students. High school teachers, on the other hand, told ACT that 35 of the 36 were important, rating only "constructing and using truth tables" lower than 3. This high school-to-college discrepancy in emphasis was detected earlier in a 1985 survey and continues today. However, as with the algebra topics, the college faculty told ACT that knowledge of geometry was important at the advanced-placement level. Of the 36 topics, 30 (83%) were rated as being at least moderately important for advanced-level freshmen.

The ACT Assessment Mathematics Test covers all of the topics college faculty rated highly and all but 4 of the topics high school teachers rated highly.

Because some of the geometry skills are valued for advanced-level freshmen and not for entry-level freshmen, the ACT Assessment Mathematics Test has a content basis for distinguishing between entry-level and advanced-level freshmen in geometry, as was seen in algebra. Geometry, as it appears in the subscores of Intermediate Algebra/Coordinate Geometry and Plane Geometry/Trigonometry, can also be used as a basis for making placement decisions in mathematics at the college level.

Probability and Statistics (P&S)

Of the 9 survey topics listed for P&S, college faculty said none of them were important enough to be rated higher than 3. High school teachers said all of them were as important as 3 or higher. This discrepancy may be an indication of the increasing emphasis being placed on P&S at the high school level, resulting from efforts such as the NCTM *Standards*. Colleges, however, do not yet find knowledge of the area to be necessary for success in entry-level or advanced-level freshmen mathematics classes.

The ACT Assessment Mathematics Test encompasses all but 2 of the topics rated 3 or higher by respondents, excluding "testing a null hypothesis" and "determining confidence intervals." The 6 remaining topics are included in the Pre-Algebra part of the test specifications. ACT's experience has been that these types of questions are currently too difficult to include on the ACT Assessment Mathematics Test. At some future time, when students are more experienced in P&S, these topics may become less difficult, and may then be included on the ACT Assessment Mathematics Test.

Trigonometry

The ACT surveys listed 6 topics under trigonometry. High school teachers told ACT that all 6 topics were important. Although college faculty said that none of the topics were important for entry-level freshmen, the faculty did find all 6 topics important for advanced-level freshmen.

The ACT Assessment Mathematics Test encompasses all 6 of the trigonometry topics. Because of the differential valuing of trigonometry for entry-level as compared to advanced-level freshmen, the Mathematics Test score and its Plane Geometry/Trigonometry subscore can be helpful in placing entering freshmen.

Calculus

Five calculus topics were listed in the surveys. High school teachers reported that all of the topics were important from their perspective. However, college faculty found none of them important for entry-level freshmen. Four of the topics—finding derivatives; determining maxima, minima, and points of inflection for functions; understanding continuity; and finding the limit of an expression—were found important for advanced-level freshmen.

The ACT Assessment Mathematics Test does not require students to demonstrate specific knowledge or skill in calculus but, in some instances, students may use calculus concepts to answer a question, such as finding the minimum value of a parabola. In general, ACT has found that calculus questions are too difficult for the general ACT-tested population, often because students have not taken calculus by the 12th grade. ACT therefore has not included calculus questions as part of the test specifications.

Table 4 lists the median ratings of the mathematics topics by level. Median ratings are the midpoints of the ratings, above and below which half of the ratings fall. For algebra at the secondary level, the respondents found, on average, that algebra was somewhat important (4 = somewhat important on the 1-to-5 scale), with a median rating of 3.9. As may be determined from reviewing the table, the median ratings of the topics by secondary teachers were higher for all content areas than were any of the ratings by

college faculty for entry-level students. For advanced-level freshmen, the median ratings were approximately as high as those for secondary students. One notable exception is probability and statistics, which was rated far lower for freshmen in either group than for secondary students.

Table 4
Median Survey Ratings for Mathematics Content Areas
(based on a 1–5 scale)

Content Area	Academic Level		
	Secondary School	Entry-level College Freshmen	Advanced-level College Freshmen
Mathematics Skills	4.0	3.4	3.9
Algebra	3.9	3.2	4.0
Geometry	3.9	2.8	3.9
Probability & Statistics	3.4	1.8	1.8
Trigonometry	3.8	2.1	3.7
Calculus	4.2	1.4	2.9

Additional Sources of Content Validity

This section describes the kinds of comparisons ACT has performed with the state curriculum frameworks, the NCTM *Standards*, and the NAEP mathematics framework. As an indicator of the potential trend of the curriculum, the NCTM *Standards* represent future directions the curriculum may take. The NAEP framework forms the basis for a national assessment often closely tied to state assessment programs and therefore closely related to state curricula. The state frameworks, in turn, depict the immediate future of the curriculum. This section also describes how teachers are involved in the development of the ACT Assessment Mathematics Test.

State Curriculum Frameworks

In 1993, ACT contacted all 50 states for copies of their most recent mathematics curriculum frameworks. ACT staff read and summarized frameworks from the responding 39 states. The summaries were used to obtain an overview of the mathematics curriculum nationally. ACT found that many states are aligning their curriculum frameworks with the NCTM *Standards*. Problem solving, for example, is the focus of many states' recommendations. Other NCTM *Standards* topics, including valuing mathematics, becoming a confident problem solver, mathematical communication, mathematical reasoning, and making mathematical connections are often emphasized also. Many states recommend that calculators and other technology be used extensively in educating students

in mathematics. Many states emphasize conceptual understanding and higher-order thinking. And many states are examining the potential of performance assessment at the classroom, school, district, and state levels and exploring the problems and benefits associated with it.

ACT found that the topic of probability and statistics is growing in importance within the curriculum, as are the related concepts of presenting data, reading charts, and interpreting graphs. Although discrete mathematics topics are also growing in importance, other topics, such as the use of logarithms as a calculation aid, have virtually disappeared from the curriculum, and factoring quadratic expressions, though seen as important for understanding relations, is receiving diminishing attention in the curriculum.

When the frameworks are compared to the survey results described in the previous section, there is a gap between what is actually taught and what is described in the curriculum frameworks. The curriculum frameworks tend to provide information about where classroom practice is headed, as opposed to what is happening today. For example, neither the high school nor the college teachers responding to ACT's survey rated as important the affective aspects of mathematics, including "becoming a confident problem solver" and "valuing mathematics." Neither did "making mathematical connections" emerge from the surveys as an important topic. The curriculum review did, however, reinforce the need for real-world contexts for mathematical problem solving, conceptual understanding, higher-order thinking, and the use of calculators, as key areas of emphasis. As the curriculum frameworks become more integrated into practice, those gaps will disappear.

NAEP Frameworks

The National Assessment of Educational Progress (NAEP) provides information about student proficiency in mathematics nationally. Based on many of the same principles as the NCTM *Standards* and many state curriculum frameworks, this assessment is being used by states as an essential part of their regular testing. Thus, its underlying curriculum frameworks both guide and reflect state curriculum frameworks. The NAEP frameworks also illustrate curriculum found to be important for students nationwide. It was therefore important for ACT to consider the NAEP mathematics framework in developing the ACT Assessment Mathematics Test. This section of this report compares the NAEP framework for mathematics with the ACT Assessment Mathematics Test specifications.

The 1994 NAEP in Mathematics was based on a model consisting of five major content strands: number sense, properties, and operations; measurement; geometry and spatial sense; data analysis, statistics, and probability; and algebra and functions. The ACT Assessment Mathematics

Test encompasses the first two strands in its Pre-Algebra category. The NAEP in Mathematics weights these strands more heavily than does the ACT Assessment Mathematics Test, however. The NAEP's "algebra and functions" are covered in the Mathematics Test areas of Elementary Algebra, Intermediate Algebra, and Trigonometry. The Mathematics Test is more heavily weighted in these areas than is the NAEP. The NAEP's "geometry and spatial sense" and "measurement" are covered in the ACT Assessment Mathematics Test in the categories of Coordinate Geometry and Geometry, and the weighting of the categories is approximately equal across the two assessments.

The NAEP in Mathematics subdivides each content strand into topics and subtopics. There is a good match between these topics and subtopics and those covered by the ACT Assessment Mathematics Test, despite such differences as the use of manipulatives and student-constructed responses on the NAEP in Mathematics but not on the ACT Assessment Mathematics Test.

The NAEP in Mathematics assesses the content strands across three ability levels: conceptual understanding, procedural knowledge, and problem solving. The three ability levels are viewed within a broader context of reasoning, communication, and connections, and are related to an overarching model of mathematical power. Unfortunately, because their categories are differently defined, the three ability levels of the NAEP and the three cognitive skill levels of the ACT Assessment Mathematics Test cannot be directly compared. Their goal is a common one, however: to determine students' ability to reason mathematically at a variety of cognitive levels.

The NAEP in Mathematics contains "families" of tasks/questions to probe the vertical nature of students' understanding, from giving a definition, to applying the defined concept in a familiar setting, to using the concept in solving a new problems that might require considerable insight, to generalizing the knowledge about the concept to a new level of understanding. Another type of "family" might measure horizontal knowledge across several content strands. Like the NAEP in Mathematics, the ACT Assessment Mathematics Test is designed to measure both the breadth and depth of student knowledge in mathematics, often in integrated ways.

The NAEP in Mathematics allows students to use calculators on all phases of the assessment that do not measure basic fact, operations, or estimation abilities. The calculators are provided by NAEP, and students are given time to practice with them. ACT also permits the use of calculators on the ACT Assessment Mathematics Test, agreeing with NAEP that calculator use is appropriate given the use of calculators that students encounter both in and beyond the mathematics classroom. ACT permits students to bring

the calculator with which they are most familiar, within certain limits dictated by test security concerns.

NCTM *STANDARDS*

The National Council of Teachers of Mathematics has advanced its recommendations for mathematics education in three volumes referred to collectively in this paper as the NCTM *Standards*. The first volume, the *Curriculum and Evaluation Standards for School Mathematics* (1989), outlines what the NCTM believes is important for students to learn and presents a vision for how students should learn. The evaluation standards focus on what to assess and how to assess it. The *Professional Standards for Teaching Mathematics* (1991) expands on that vision of how students should be taught and prescribes principles for the development and evaluation of teachers. The *Assessment Standards for School Mathematics* (1995) expands on the evaluation standards from the first volume.

The theme of the NCTM *Standards* is that students should learn to think mathematically. This theme is reflected in the earliest philosophical underpinnings of the ACT Assessment Mathematics Test, a test designed to measure what students can do with what they know—their mathematical reasoning—rather than concentrating on recall of formulas, knowledge of facts, or definitions of terms.

All of the topics recommended in the NCTM *Standards* appear in the content specifications for the ACT Assessment Mathematics Test. Over the last few years, many of the topics that the NCTM *Standards* recommend be deemphasized have accordingly decreased in frequency on the ACT Assessment Mathematics Test, and topics that the NCTM *Standards* recommend receive increased emphasis have grown in frequency. However, the ACT Assessment Mathematics Test does not extend to the full depth of the NCTM *Standards*-recommended curriculum. For example, the probability questions on the ACT Assessment Mathematics Test currently focus on more basic concepts of probability, while the NCTM *Standards* recommend a broader range of probability topics. As the curriculum continues to grow and students master more advanced probability concepts, the ACT Assessment Mathematics Test will measure those skills.

The NCTM *Standards* discuss problem solving, mathematical communication, reasoning, and making mathematical connections. The ACT Assessment Mathematics Test addresses all of these key areas. In problem solving, the test strives for true-to-life contexts that the students will see as important mathematically. The ACT Assessment Mathematics Test measures communication skills largely in the areas of mathematical reading skills and recognizing appropriate written mathematics. Mathematical reasoning is the foundation of the ACT Assessment Mathematics Test: there is significant agreement between the NCTM

Standards and the ACT Assessment Mathematics Test in this area. As to making mathematical connections, many problems on the ACT Assessment Mathematics Test cross mathematics-topic boundaries; some of these questions require an integrated knowledge of several topics, or connecting mathematics with the everyday world. In addition to these goals, the NCTM *Standards* call for integrating mathematics with other content areas. The ACT Assessment Mathematics Test presents its problems in real-world contexts; however, to avoid confusion over what is being measured, the test does not require specific knowledge in these other content areas in order to be able to answer the questions correctly.

ACT policy emphasizes the need for multiple sources of assessment information for making decisions about students. Similarly, the NCTM *Standards* also call for the use of multiple types of assessments: multiple-choice, short-answer, discussion, or open-ended questions; structured or open-ended interviews; homework; projects; journals; essays; dramatizations; and class presentations. Both organizations have a common goal—that of basing decisions about students on as much information as possible about what they know and can do.

The NCTM *Standards* recommend that calculators and computers be available to students at all times. Within limits of test security, students may bring whatever calculator they wish to use for the ACT Assessment Mathematics Test. All questions, however, can be answered without a calculator.

The goal of ACT, and of the NCTM *Standards*, is to help students be successful in mathematics throughout and after high school. Mathematical reasoning is the common vehicle. Because the ACT Assessment Mathematics Test is designed to measure mathematics skills that are important for success in college, and many of these skills are also reflected in the NCTM *Standards*, they share a common goal and theme.

Textbook Review

To survey what kinds of materials teachers are using in the high school classroom, ACT reviews a wide variety of mathematics textbooks. There are currently three types of high school mathematics textbooks—traditional, integrated, and NCTM *Standards*-based.

The content and presentation of the traditional textbook series reflect a lecture-based approach, accompanied by a set of problems focusing on a particular skill, with some features added to meet NCTM *Standards*. Some of these features are geometric modeling to clarify algebraic concepts and suggested activities to use a calculator or computer; most are additions to the existing approach. The practice exercises contain sets of problems that are very similar in nature, some of which may include application problems. (A variation on the "traditional" type contains traditional

content but is organized by problem sets intended to promote continual practice.)

In the early 1990s there was a push to fully integrate two years of algebra with one of geometry. Several new textbook series were then published. These integrated series have not yet been widely accepted even though several states now require such integration. Teachers have indicated that time and training are needed to make these textbook changes.

The most recent development has been to incorporate the NCTM *Standards* into mathematics textbooks. These new series expand and reorganize the mathematical content while integrating technology. Influenced by the NCTM *Standards*, the content and presentation in these textbooks are notably different from those of traditional series. More attention is given to real-world applications and to the understanding of underlying mathematical principles. The content integrates data interpretation, probability, statistics, and discrete mathematics topics into all mathematics courses. The content is also introduced in a new order of presentation. For example, functions are presented in the first year of algebra instead of the second; and matrices appear in some first-year algebra textbooks in the NCTM *Standards*-based series.

A wide variety of textbooks appear in high school mathematics classrooms. ACT uses textbooks to corroborate survey evidence about the timing of the introduction of mathematics topics to students. The curriculum is not static, however—both the NCTM *Standards* and evolving technologies are greatly influencing the mathematics taught and the way it is taught. ACT continually monitors these developments and takes them into account in developing the ACT Assessment Mathematics Test.

Involvement of U.S. Mathematics Teachers in the Test Development Process

Production of Test Questions

ACT has three goals in selecting question writers to develop its tests:

- to work with classroom teachers who have day-to-day knowledge of what is happening in mathematics classrooms nationwide;
- to achieve balanced perspectives of high school teachers and college faculty; and
- to ensure that the question writers represent the diversity of the population of the United States with respect to ethnic background, gender, and geographic location.

ACT recruits question writers who represent a cross section of educators within the United States and to reflect the diversity of the population served by the ACT Assessment. Writers are trained in sound question writing practices by ACT staff, and they work closely with ACT test specialists, who assist them in producing questions of high quality that meet the test specifications.

Review of Test Questions

High school and college mathematics teachers are also directly involved in the process of ACT Assessment test-question review. They are involved in two kinds of review: fairness and content accuracy. Along with other persons sensitive to issues of test and question fairness, teachers are a part of the fairness review process. They examine all prospective test questions and passages for ethnic and gender fairness and representativeness.

High school and college mathematics teachers participate in verifying the content accuracy of the ACT Assessment as well. They ensure, by reviewing all prospective test questions, that the test materials are accurate and representative of the curriculum, as well as grade appropriate.

A minimum of 20 independent reviews are performed on each test question during the 2½ years before it appears on a national ACT Assessment test. The many reviews help ensure that each student's level of achievement is accurately and fairly measured. The teachers' perspective is vital in every stage of the test development process.

ACT Research

Course Placement Research

If the purpose of the ACT Assessment is to measure that which is taught in high school that is important for college success, then it follows that the relationship of test performance to college grade point average is a valid criterion of the ACT Assessment's success. ACT conducts considerable research into this relationship and other associated topics to ensure that the tests measure what ACT says they measure for the purposes for which the tests are being used (validity) and that the tests will be highly accurate in their measurement (reliability). Research on predictive validity, for example, examines how well the ACT Assessment can predict college success. Through its Course Placement Service, ACT works with individual colleges and universities to establish the relationship between the ACT Assessment scores and high school grade point average and college success, based on students' performance in the courses in which they are placed. ACT's purpose is to demonstrate that the ACT Assessment predicts college success accurately and effectively.

Colleges use ACT Assessment scores alone or in combination with other factors in making course placement decisions. Among participants in ACT's Course Placement Service, about 92% have used the ACT Assessment to help decide whether students should enroll in a standard course or in some kind of remedial course in areas of language arts, science, and mathematics. About 8% of the participating schools have used ACT Assessment scores to decide between a standard course and some kind of advanced course.

Mathematics Test scores are the most commonly used of the single-test scores in placement decisions. (Mathematics Test scores are often also used in combination with Science Reasoning scores.) In making placement decisions, Mathematics Test scores are most often used to assign students to college mathematics courses at the following levels: pre-algebra, intermediate algebra, college algebra, and calculus. ACT's research shows that placement in mathematics has a high level of success: on average at least 65% of students are accurately placed based on their Mathematics Test score, either alone or with other factors. Results of this same research indicate that the median accurate placement rate in intermediate algebra, for example, was approximately 69%. If no Mathematics Test score had been used in making the decision and all students had been permitted to take the course, the expected accuracy rate would have been 41%. This is an increase in accuracy of 28% over what would have been expected without using the ACT Assessment Mathematics Test score. Table 5 presents the data for other mathematics courses.

Table 5
ACT Assessment Mathematics Test Cutoff Scores and Decision-Based Validity Statistics for Course Placement

Course Type	No. of Institutions	Median Cutoff Score	Median Accuracy Rate	Median Increase in Accuracy Rate
Intermediate Algebra	15	22	69	28
College Algebra	44	23	71	28
Statistics/Probability	4	23	77	35
Pre-Calculus	14	25	72	36
Trigonometry	16	26	71	33
Calculus	34	26	78	47

Such increases in accuracy for course placement support the usefulness of the ACT Assessment Mathematics Test for college placement. This is a direct result of the close relationship between the ACT Assessment Mathematics Test and the mathematics curriculum nationally.

Course Work Research

An additional area of validity research addresses the impact of course work on test scores. ACT conducts considerable validity research that seeks confirmation that the ACT Assessment is a sound measure of educational development. Results of this research repeatedly confirm that students who meet core course requirements (4 years of English, and 3 years each of mathematics, social studies, and science) will perform better on the ACT Assessment than those who do not. ACT's research has clearly demonstrated that students taking or planning to take more mathematics courses perform better than those who take or plan to take fewer. Moreover, those students who take more mathematics courses in high school score higher than those who take fewer (1-2) mathematics courses.

In a 1992 study, ACT researchers used a representative sample of 40,000 ACT Assessment-tested 1989-90 high school juniors and seniors from 400 high schools. The researchers found that, for each additional mathematics course taken, students' Mathematics Test scale scores increased, on average, by 1.27 scale score points. This finding—that students who have taken and/or plan to take more mathematics courses (and in the fall of grade 12 students can plan for at most only one semester of high school courses) achieve higher Mathematics Test scores than those who do not—confirms the close relationship between the ACT Assessment Mathematics Test and the mathematics curriculum at the high school level, and further documents the curriculum-based nature of the test.

Summary and Conclusions

The content of the ACT Assessment Mathematics Test is based on a complex analysis of the high school curricula and what colleges require for success in entry-level and advanced-level courses. Results of ACT's studies confirm that the ACT Assessment Mathematics Test is firmly grounded in the curriculum and measures content and skills students learn in high school that are important for college success.

ACT's national curriculum surveys, reviews of state curriculum frameworks, and monitoring of national standards form the basis for the knowledge/skill continuum underlying the test specifications. Mathematics educators, notably high school teachers, curriculum specialists, and college faculty, all play important roles in all phases of ACT Assessment Mathematics Test development. Finally, ACT research provides strong evidence that the test predicts college success and is useful for accurate placement of students in college courses.

The relationship between the ACT Assessment Mathematics Test and the high school and college-entry mathematics curriculum is a close one. Because the curriculum is dynamic, ACT continuously monitors national

developments related to curriculum standards, changes in state curriculum frameworks, new developments in technology and textbooks, and other factors that affect the curriculum taught in the schools. ACT also continues to explore new assessment alternatives that offer students additional ways of demonstrating what they know and can do in mathematics. Finally, and most importantly, ACT continues to rely on the critical contributions that mathematics educators at the secondary and postsecondary levels make to all phases of the test development process.

Appendix

Mathematics Curriculum Survey Results: Mean Rating and Rank for Each Content/Skill Topic by Level

		Secondary Responses				Postsecondary Responses			
						For Entry-Level Freshmen		For Advanced-Level Freshmen	
	Skill or Content Surveyed	% Responding Taught before Grade 12	% Responding Not Taught	Rank	Mean Rating	Rank	Mean Rating	Rank	Mean Rating
	Mathematics Skills								
1	Choosing an appropriate method for problem solving (e.g., mental, paper and pencil, calculator or estimation)	97	3	4	4.3	4	3.7	3	4.1
2	Solving problems posed in real-world settings	98	2	2	4.4	9	3.4	11	3.8
3	Applying real-world constraints when interpreting the solution to a problem	95	4	8	4.1	10	3.4	13	3.7
4	Applying speed and accuracy in problem solving	94	6	16	3.6	14	3.2	14	3.6
5	Constructing an inductive proof	89	7	21	3.0	20	2.0	20	2.8
6	Constructing a deductive proof	94	5	20	3.3	19	2.1	19	3.1
7	Recognizing generalizations of mathematical ideas	97	3	13	4.0	12	3.3	7	4.1
8	Recognizing and using patterns to solve problems	98	1	6	4.1	1	3.8	2	4.2
9	Applying mathematical ideas to new contexts	95	4	7	4.1	6	3.6	4	4.1
10	Applying theorems	96	3	15	3.8	15	3.1	8	4.0
11	Recognizing when essential data are missing	96	4	12	4.0	7	3.4	10	3.8
12	Using estimation	98	2	11	4.0	13	3.3	15	3.6
13	Finding optimal solutions to problems	85	9	17	3.6	17	2.8	16	3.5
14	Formulating new patterns or structures	85	10	18	3.6	16	2.9	17	3.5
15	Reading and interpreting graphs, charts, and other data representations	98	2	3	4.3	2	3.7	6	4.1
16	Planning and carrying out a strategy for solving multi-step problems	98	2	1	4.4	3	3.7	1	4.4
17	Performing basic operations with a calculator	98	2	5	4.1	8	3.4	12	3.8
18	Performing statistical operations with a calculator	68	22	19	3.5	21	1.9	21	2.0
19	Graphing and exploring the nature of equations using a graphing calculator	77	12	14	3.9	18	2.5	18	3.3
20	Communicating mathematically (written)	92	7	10	4.1	5	3.6	5	4.1
21	Communicating mathematically (oral)	92	6	9	4.1	11	3.3	9	3.9

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	Skill or Content Surveyed	Secondary Responses: % Responding Taught before Grade 12	Secondary Responses: % Responding Not Taught	Secondary Responses: Rank	Secondary Responses: Mean Rating	Postsecondary Responses: For Entry-Level Freshmen: Rank	Postsecondary Responses: For Entry-Level Freshmen: Mean Rating	Postsecondary Responses: For Advanced-Level Freshmen: Rank	Postsecondary Responses: For Advanced-Level Freshmen: Mean Rating
	Algebra								
1	Understanding domain and range	98	2	**24**	3.9	**22**	3.2	**14**	4.4
2	Working with series and sequences(arithmetic and geometric)	86	2	**35**	3.6	**40**	2.2	**36**	3.3
3	Finding the sum of an infinite geometric series	73	6	**40**	3.3	**41**	2.0	**41**	3.1
4	Evaluating algebraic expressions by substitution	99	1	**5**	4.4	**5**	4.0	**7**	4.5
5	Working with percent (simple interest, tax, sale markdowns, etc.)	97	3	**10**	4.3	**19**	3.3	**39**	3.2
6	Understanding scientific notation	98	2	**29**	3.7	**25**	3.1	**29**	3.6
7	Using sets and set notation(including Venn diagrams)	86	12	**45**	3.0	**32**	2.7	**40**	3.2
8	Understanding sigma notation	71	6	**38**	3.5	**36**	2.5	**31**	3.5
9	Understanding the concept of a function	98	2	**12**	4.2	**12**	3.6	**3**	4.7
10	Solving equations and inequalities involving absolute value	98	2	**28**	3.8	**24**	3.2	**22**	4.1
11	Solving linear equations in one variable	98	2	**1**	4.6	**2**	4.2	**1**	4.7
12	Recognizing the slope-intercept and standard form of a linear equation	98	2	**7**	4.3	**6**	3.9	**5**	4.6
13	Determining the inverse of a function	92	3	**31**	3.6	**31**	2.8	**24**	4.1
14	Solving a linear inequality in one variable	98	2	**16**	4.1	**16**	3.5	**17**	4.3
15	Using the quadratic formula	98	2	**8**	4.3	**11**	3.6	**11**	4.4
16	Approximating roots of polynomial and rational equations	85	5	**32**	3.6	**37**	2.4	**32**	3.4
17	Understanding the compositions of functions	87	4	**33**	3.6	**28**	2.9	**8**	4.5
18	Working with parametric equations	48	16	**44**	3.2	**43**	2.0	**34**	3.3
19	Using the discriminant	92	3	**37**	3.5	**38**	2.4	**38**	3.2
20	Working with exponential and logarithmic functions	88	3	**26**	3.9	**29**	2.9	**15**	4.3
21	Solving quadratic equations by factoring	98	2	**17**	4.0	**15**	3.5	**16**	4.3
22	Solving quadratic inequalities	94	4	**34**	3.6	**33**	2.7	**30**	3.5
23	Solving a system of two linear equations in two variables	99	1	**13**	4.2	**23**	3.2	**20**	4.1
24	Understanding the absolute value of a number	99	1	**15**	4.2	**7**	3.9	**4**	4.6

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	Skill or Content Surveyed	Secondary Responses: % Responding Taught before Grade 12	Secondary Responses: % Responding Not Taught	Secondary Responses: Rank	Secondary Responses: Mean Rating	Postsecondary Responses: For Entry-Level Freshmen: Rank	Postsecondary Responses: For Entry-Level Freshmen: Mean Rating	Postsecondary Responses: For Advanced-Level Freshmen: Rank	Postsecondary Responses: For Advanced-Level Freshmen: Mean Rating
25	Performing operations with integer exponents	98	1	6	4.3	4	4.0	2	4.7
26	Factoring an expression into prime components	97	2	23	3.9	21	3.3	27	3.9
27	Working with rational exponents	96	2	19	4.0	17	3.4	10	4.4
28	Understanding ratio and proportion	99	1	3	4.5	9	3.7	23	4.1
29	Converting fractions to decimals and decimals to fractions	97	3	11	4.2	14	3.6	28	3.8
30	Performing operations with rational expressions	98	1	18	4.0	13	3.6	18	4.3
31	Understanding base and modular arithmetic	45	50	46	2.5	45	1.8	46	2.0
32	Finding common factors and common multiples of numbers	96	3	22	3.9	27	3.1	33	3.4
33	Finding determinants	50	6	42	3.3	44	1.8	44	2.3
34	Performing matrix addition and multiplication	71	11	41	3.3	46	1.7	45	2.2
35	Understanding the order of real numbers on the number line	98	1	4	4.4	3	4.1	9	4.4
36	Working with complex numbers	93	4	30	3.7	42	2.0	43	2.8
37	Performing addition, subtraction, multiplication, and division of real numbers	97	3	2	4.5	1	4.5	6	4.5
38	Implementing remainder and factor theorems	78	5	39	3.4	39	2.3	42	3.1
39	Performing polynomial long division	94	3	43	3.2	34	2.6	35	3.3
40	Factoring polynomial expressions	97	1	21	4.0	20	3.3	19	4.1
41	Determining the roots of polynomial equations	93	2	20	4.0	30	2.9	26	3.9
42	Simplifying algebraic expressions	98	1	9	4.3	8	3.8	12	4.4
43	Performing addition, subtraction, and multiplication of polynomials	98	2	14	4.2	10	3.7	13	4.4
44	Performing operations with radical expressions	98	1	25	3.9	18	3.3	21	4.1
45	Simplifying radical expressions	98	2	27	3.8	26	3.1	25	4.0
46	Knowing approximations of square roots and irrational numbers	92	7	36	3.5	35	2.6	37	3.3
	Calculus								
1	Finding derivatives	13	15	1	4.3	3	1.5	4	3.0
2	Finding integrals	8	17	2	4.2	5	1.4	5	2.9

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	Skill or Content Surveyed	Secondary Responses				Postsecondary Responses			
						For Entry-Level Freshmen		For Advanced-Level Freshmen	
		% Responding Taught before Grade 12	% Responding Not Taught	Rank	Mean Rating	Rank	Mean Rating	Rank	Mean Rating
3	Determining maxima, minima, and points of inflection for functions	21	12	3	4.2	2	1.5	3	3.0
4	Understanding continuity	18	14	5	4.0	1	1.5	1	3.1
5	Finding the limit of an expression	21	12	4	4.1	4	1.4	2	3.1
	Geometry								
1	Locating points in the coordinate plane	99	2	3	4.2	1	4.0	1	4.8
2	Graphing on the number line	99	2	14	4.0	2	4.0	5	4.5
3	Graphing rational functions	96	3	20	3.7	22	2.6	18	3.9
4	Using the Pythagorean theorem	96	4	1	4.5	7	3.6	4	4.5
5	Measuring and constructing right, acute, and obtuse angles	97	2	26	3.6	24	2.6	30	3.1
6	Using manipulatives	61	35	28	3.4	32	2.2	36	2.1
7	Finding the distance between two points	97	3	18	3.9	11	3.2	11	4.1
8	Understanding the concepts and relationships for plane and solid geometric figures	88	12	24	3.7	31	2.3	27	3.3
9	Working with graphs in three-space	86	14	35	3.0	34	1.9	32	2.9
10	Determining the volume of a prism, pyramid, cylinder, and cone	87	10	27	3.5	30	2.4	29	3.1
11	Constructing graphical solutions to systems of equations and inequalities	86	11	19	3.9	17	2.9	23	3.7
12	Knowing the ratio of sides in 45-45-90 triangles and 30-60-90 triangles	97	2	11	4.1	28	2.5	16	4.0
13	Working with vectors in a plane	64	21	30	3.4	35	1.8	34	2.6
14	Using different measurement systems and converting between them	91	7	34	3.2	27	2.5	31	2.9
15	Constructing and using truth tables	66	14	36	2.7	36	1.8	35	2.2
16	Understanding transformations	97	3	31	3.4	23	2.6	24	3.5
17	Understanding the properties of triangles	98	2	5	4.2	15	3.0	14	4.0
18	Working with inscribed and circumscribed polygons and circles	95	4	29	3.4	33	2.1	33	2.9
19	Working with equations of parallel and perpendicular lines	98	2	17	3.9	12	3.2	13	4.1

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	Skill or Content Surveyed	Secondary Responses: % Responding Taught before Grade 12	Secondary Responses: % Responding Not Taught	Secondary Responses: Rank	Secondary Responses: Mean Rating	Postsecondary Responses: For Entry-Level Freshmen: Rank	Postsecondary Responses: For Entry-Level Freshmen: Mean Rating	Postsecondary Responses: For Advanced-Level Freshmen: Rank	Postsecondary Responses: For Advanced-Level Freshmen: Mean Rating
20	Working with equations of parabolas, circles, ellipses, and hyperbolas	87	6	23	3.7	21	2.7	20	3.8
21	Finding circumference and arc length of a circle	85	4	25	3.7	16	3.0	22	3.7
22	Understanding the concept of angle and angle measure	97	3	6	4.2	14	3.1	12	4.1
23	Understanding parallel lines and transversals	98	2	33	3.3	8	3.5	9	4.1
24	Graphing parabolas, circles, ellipses, and hyperbolas	97	3	22	3.7	20	2.7	17	3.9
25	Working with congruent and similar triangles	88	6	9	4.1	19	2.8	21	3.8
26	Recognizing the relationship between a function and its graph	98	2	10	4.1	4	3.7	3	4.6
27	Knowing the properties of a circle: radius, diameter, circumference, chord, and tangents	98	2	12	4.0	13	3.1	10	4.1
28	Finding the slope of a line	98	2	2	4.3	3	3.9	2	4.6
29	Understanding perpendicular lines	98	2	13	4.0	6	3.6	6	4.3
30	Graphing linear equations in two variables	86	12	8	4.1	9	3.5	8	4.2
31	Determining a locus of points	97	3	16	3.9	25	2.6	25	3.5
32	Constructing geometric figures	94	4	7	4.1	26	2.6	28	3.2
33	Understanding lines, segments, and rays	98	2	15	3.9	10	3.4	15	4.0
34	Graphing translations	98	2	32	3.3	29	2.4	26	3.3
35	Using the midpoint formula	98	2	21	3.7	18	2.9	19	3.9
36	Identifying plane geometric figures	97	3	4	4.2	5	3.6	7	4.3
	Trigonometry								
1	Solving trigonometric equations	71	5	3	3.9	4	2.2	3	3.7
2	Using trigonometric identities	66	5	5	3.7	2	2.3	2	4.0
3	Working with right triangle trigonometry	84	4	1	4.3	1	2.4	1	4.0
4	Using addition formulas for sine and cosine	64	6	6	3.4	5	2.1	5	3.5
5	Using the law of sines and law of cosines	70	5	2	4.0	6	2.1	6	3.3
6	Graphing trigonometric functions, including amplitude, period, and phase shift	64	6	4	3.9	3	2.3	4	3.6
	Probability and Statistics								
1	Understanding the concept of probability	81	9	2	3.9	1	2.1	1	2.2

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	Skill or Content Surveyed	Secondary Responses: % Responding Taught before Grade 12	Secondary Responses: % Responding Not Taught	Secondary Responses: Rank	Secondary Responses: Mean Rating	Postsecondary Responses: For Entry-Level Freshmen: Rank	Postsecondary Responses: For Entry-Level Freshmen: Mean Rating	Postsecondary Responses: For Advanced-Level Freshmen: Rank	Postsecondary Responses: For Advanced-Level Freshmen: Mean Rating
2	Using counting techniques (including permutations and combinations)	72	13	4	3.6	2	2.1	2	2.1
3	Understanding mutually exclusive, dependent and independent events	64	18	5	3.4	3	2.0	3	2.1
4	Collecting and representing (e.g., circle graphs and frequency distributions) data	77	14	3	3.8	4	2.0	4	2.0
5	Understanding linear regression and correlation	44	36	6	3.3	8	1.6	6	1.8
6	Finding the variance and standard deviation of data	43	36	7	3.2	6	1.7	7	1.7
7	Finding the mean, median, and mode	84	10	1	4.0	5	1.9	5	2.0
8	Testing a null hypothesis	30	57	8	3.1	9	1.6	9	1.5
9	Determining confidence intervals	24	58	9	3.0	7	1.6	8	1.6

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